Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

Sample WACE Examination 2016

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

 2014/10404

Section One: Calculator-free **35% (49 marks)**

Question 1 **(6 marks)**

(a) The shopping bus for senior citizens has been operating for three months. The increase in the number of senior citizens using the bus is described by the recursive equation:

$$T_{n+1} = T_n + 12, \quad T_3 = 43$$

(i) Determine the number of senior citizens who will use the bus in the fourth month. (1 mark)

Solution
$43 + 12 = 55$
Specific behaviours
✓ calculates the correct answer

(ii) Determine T_1 and describe what it represents. (2 marks)

Solution
$43 - 12 - 12 = 19$ This is the number of people using the bus in the first month
Specific behaviours
✓ calculates the correct answer ✓ describes what T_1 represents

(b) The number of recycling bins emptied monthly was growing at a rate of nine extra bins each month. In the first month of last year there were 550 bins emptied.

(i) Display the terms of the sequence for the number of bins emptied each month for the first 4 months of last year. (1 mark)

Solution
550, 559, 568, 577
Specific behaviours
✓ displays the correct sequence

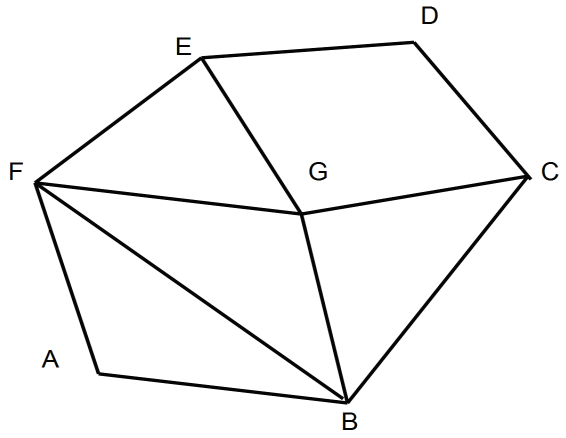
(ii) Deduce a non-recursive rule to describe T_n, the n^{th} term. (2 marks)

Solution
$T_n = 9n + 541$
Specific behaviours
✓ evaluates the common difference ✓ sets up the correct linear relation

(iii) Determine T_{50}. (1 mark)

Solution
$T_{50} = 9 \times 50 + 541 = 991$
Specific behaviours
✓ evaluates T_{50}

Question 2 **(7 marks)**



(a) Verify that Euler's formula works for this network. (2 marks)

Solution
$v = 7, r = 6, e = 11$ $v + r - e = 2$ $7 + 6 - 11 = 2$
Specific behaviours
✓ calculate v, r and e correctly ✓ applies Euler's rule correctly

(b) Determine the degree of each vertex in the network above and use the table to record your results. (2 marks)

Vertex	A	B	C	D	E	F	G
Degree	2	4	3	2	3	4	4

Solution
values as shown in red in table
Specific behaviours
✓ determines five correct values ✓ determines all seven values correct

(c) This network forms a semi-Eulerian trail.

(i) What property of the vertices supports the claim that this network forms a semi-Eulerian trail? (1 mark)

Solution
There are only two vertices of odd degree
Specific behaviours
✓ identifies a semi-Eulerian trail

(ii) Explain what is meant by a semi-Eulerian trail. (1 mark)

Solution
A trail in which each edge is visited exactly once
Specific behaviours
✓ defines a semi-Eulerian trail correctly

(iii) Describe a semi-Eulerian trail by listing the vertices of the trail in the order visited. (1 mark)

Solution
e.g. EGB FGC BAF EDC
Specific behaviours
✓ lists one set of vertices in the order visited, correctly

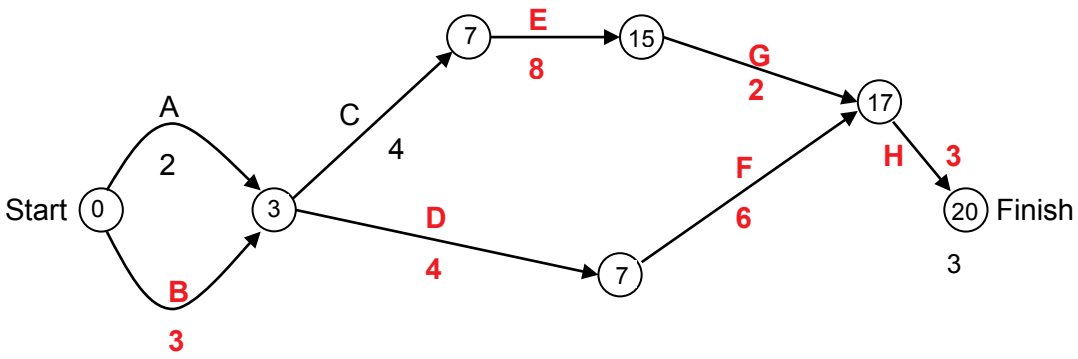
Question 3 **(4 marks)**

Activities 'A', 'B', 'C' ..., 'H' are required to build a small extension to an existing house. The estimated completion times (in weeks) for these activities are shown in the table below.

Activity	Completion time (weeks)	Predecessor(s)
A	2	-
B	3	-
C	4	B, A
D	4	B, A
E	8	C
F	6	D
G	2	E
H	3	F, G

(a) Complete the project network by labelling the arcs. (1 mark)

Solution



Specific behaviours

✓ completes project network labelling arcs correctly

(b) State the critical path. (1 mark)

Solution

BCEGH

Specific behaviours

✓ correctly determines critical path

(c) State the minimum number of weeks required to build the extension. (1 mark)

Solution

20 weeks

Specific behaviours

✓ correctly determines minimum completion time based on part (b)

(d) What is the maximum number of weeks by which CEG can be reduced before there is a change to the critical path? (1 mark)

Solution

Reduces path CEG to 10 weeks, a drop of 4 weeks on the original path

Specific behaviours

✓ changes CEG to 10 weeks to compare time reduction

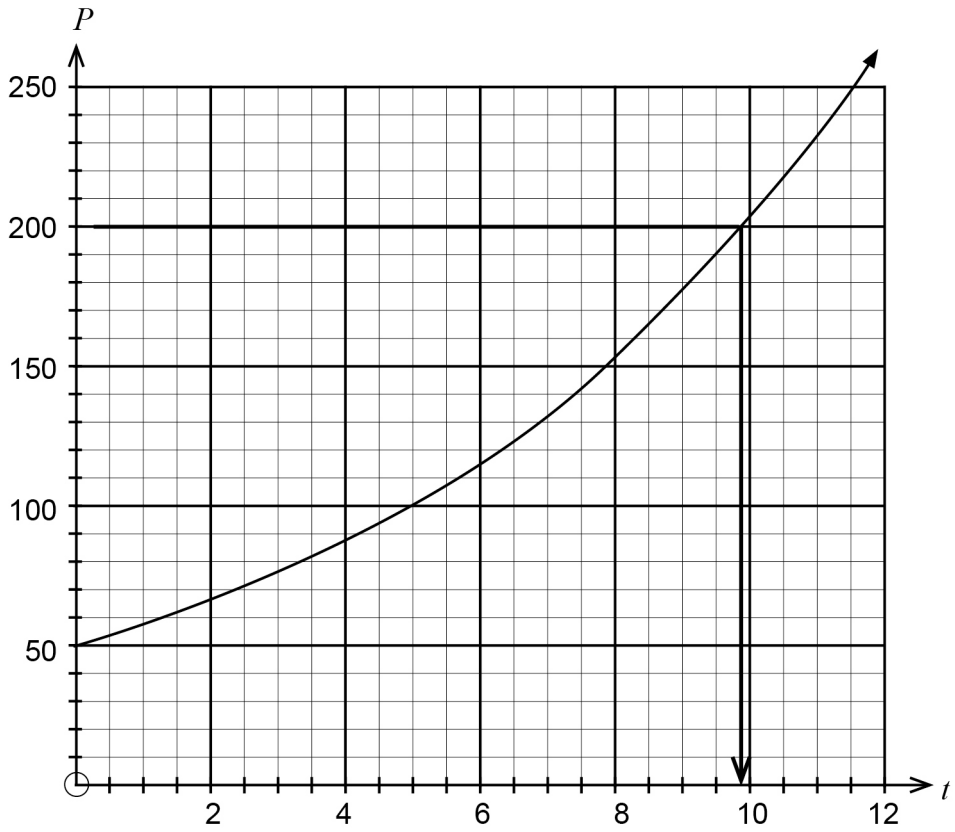
Question 4 **(6 marks)**

Scientists were trying to increase the population P of a rare species of fish, so they placed a number of fish in a small lake and monitored the population monthly.

(a) A graph of the fish population is shown below. Use the graph to determine the

(i) number of fish originally placed in the lake. (1 mark)

(ii) month during which the population of the fish first exceeded 200. (1 mark)



Solution
(i) 50 (ii) First exceeds in 10th month
Specific behaviours
✓ reads $P = 50$ from graph ✓ correctly states the tenth month

(b) The scientists discovered that the population of the fish, P, increased according to the rule:

$P = 50 \times 1.15^t$, where t was the time in months after the fish were placed in the lake.

Complete the next **two** rows of the table below. (1 mark)

Time (number of months after fish placed in the lake)	Calculation	Population
1	50 × 1.15	58
2	50 × 1.15 × 1.15	66
3	**50 × 1.15 × 1.15 × 1.15**	**76**
4	**50 × 1.15 × 1.15 × 1.15 × 1.15**	**87**

Solution
As shown in table in bold
Specific behaviours
✓ completes calculations correctly

(c) Use the information presented in part (b) to determine the rate of growth of the fish population. (2 marks)

Solution
15% per month
Specific behaviours
✓ states 15% ✓ identifies per month

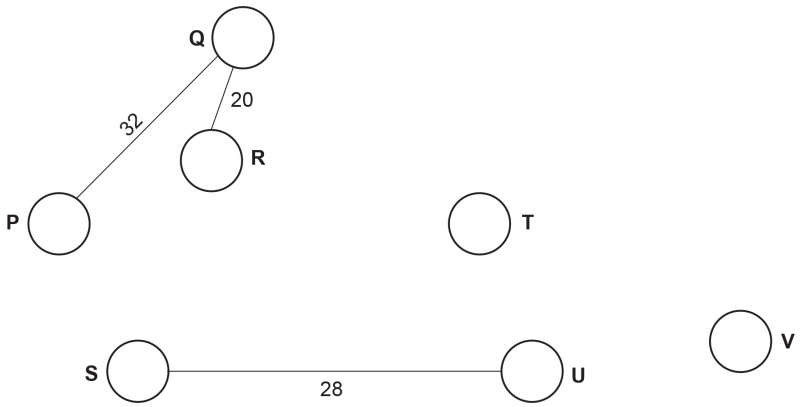
(d) Describe the long-term growth of the fish population in the lake. (1 mark)

Solution
All other aspects being fair (size of lake, food supply) the population will increase exponentially. The rule has a positive ratio (r value) **or** the graph has an increasingly positive slope.
Specific behaviours
✓ identifies population increases exponentially

Question 5 **(6 marks)**

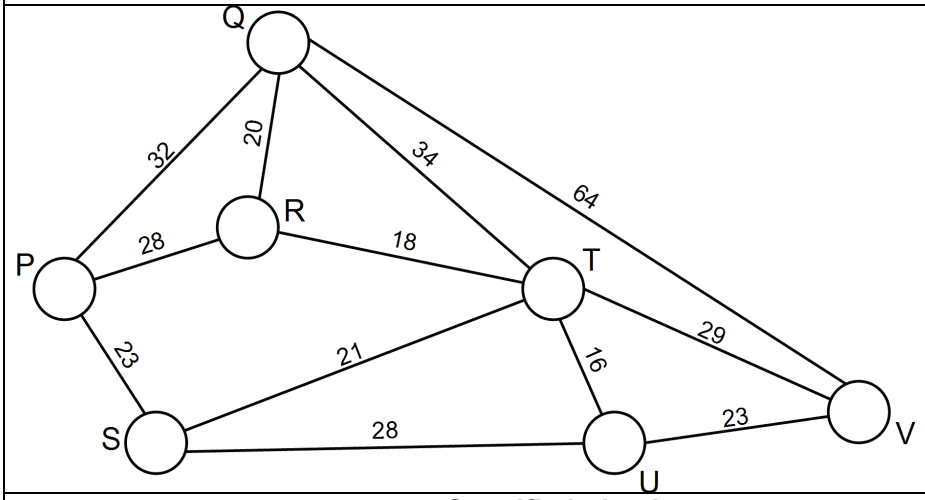
The table below shows travel times (in minutes) along lines of a rail network connecting stations at ‘P’, ‘Q’, ‘R’, ‘S’, ‘T’, ‘U’ and ‘V’. The diagram (which is not drawn to scale) shows the positions of the stations.

	P	Q	R	S	T	U	V
P	-						
Q	32	-					
R	28	20	-				
S	23	-	-	-			
T	-	34	18	21	-		
U	-	-	-	28	16	-	
V	-	64	-	-	29	23	-



(a) Complete the diagram by drawing all the rail lines in the network. Label the lines with the travel times. (2 marks)

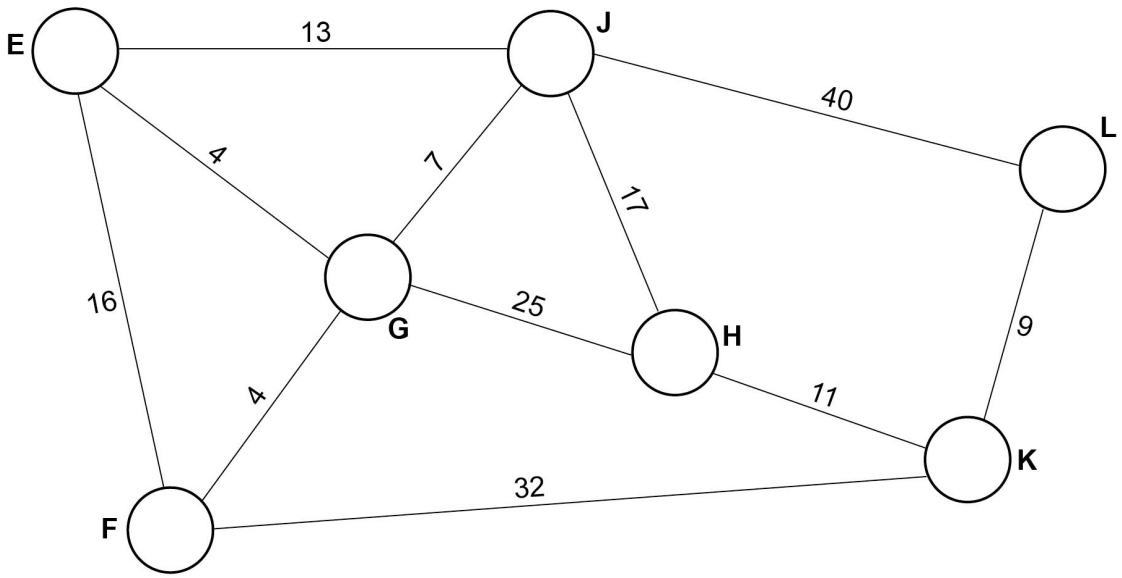
Solution



Specific behaviours

✓ correctly draws and labels at least four lines
✓ correctly draws and labels all lines

(b) A second rail network connecting stations ‘E’, ‘F’, ‘G’, ‘H’, ‘J’, ‘K’ and ‘L’, with travel times between the stations given in minutes, is shown below.



A maintenance worker based at Station E needs to travel to each other station in turn, without visiting any station more than once. The worker will finish the trip at a different station to the origin E.

(i) What is the name given to such a path which each vertex is included only once? (1 mark)

(ii) One possible path is E F G J H K L. What is the travel time for this path? (1 mark)

(iii) Use the letters of the vertices to complete two other possible paths. (2 marks)

Path 1: E G

Path 2: E J

Solution
(i) Hamiltonian Path (ii) 64 minutes (iii) Path 1: E G **H J L K F**, Path 2: E J **H G F K L**
Specific behaviours
✓ identifies Hamilton path ✓ calculates travel time correctly ✓✓ determines two other possible paths for the given vertices (one mark each)

Question 6 **(6 marks)**

Some students were studying statistics and were involved in various projects to collect and analyse data.

(a) One student, Amy, wanted to find an answer to the question:
'Do students in my school tend to travel a long way to school?'
Amy planned to survey ten Year 12 students in her class.

(i) Write one question that she could ask the students so that she could gather data to answer her question. (1 mark)

(ii) State one way in which she could improve her data collection. (1 mark)

Solution
(i) Any reasonable question, such as 'How many kilometers is it between your home and school?'
(ii) Ask a greater number of students or students from different classes
Specific behaviours
✓ asks a question which would allow the appropriate data to be collected
✓ suggests a larger or more diverse sample

(b) Another student, Matthew, was investigating the means by which students travelled to school. He gathered the following data, which came from the 2013 survey in the CensusAtSchool Australia project.

Method of travel to school (%)

Method of travel	WA	Aust
Bicycle	5.9	3.4
Boat/Ferry	0.5	0.5
Bus	26.2	25.4
Car	41.1	46
Skateboard/Scooter/Rollerblades	2.7	1.3
Train/Tram	2.4	4.8
Walk	20.5	18.2
Other	0.6	0.4
Total	**100**	**100**

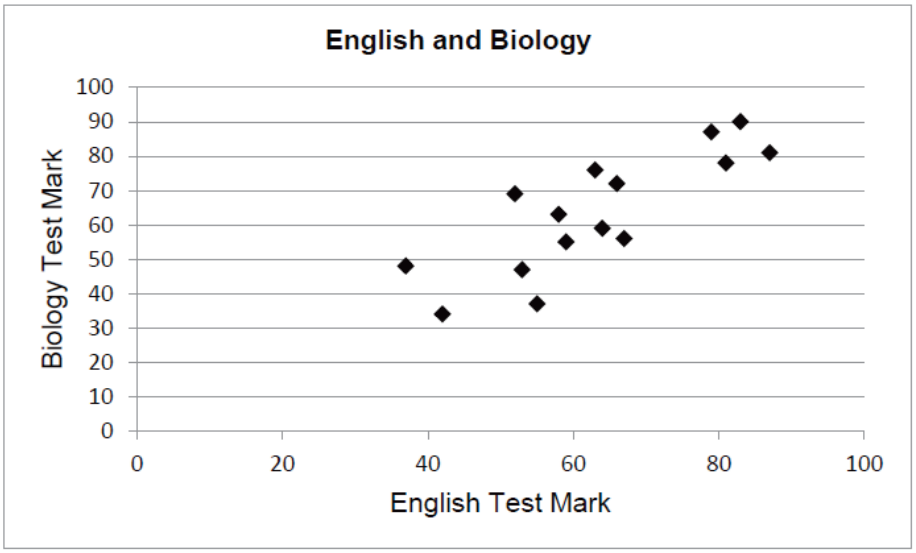
(i) What type of graph would be suitable to show a comparison between the different methods of travel within Western Australia? (1 mark)

(ii) Matthew concluded that the low percentage of students riding to school was due to the roads being too dangerous. Suggest another reason that might account for the low percentage of students riding to school. (1 mark)

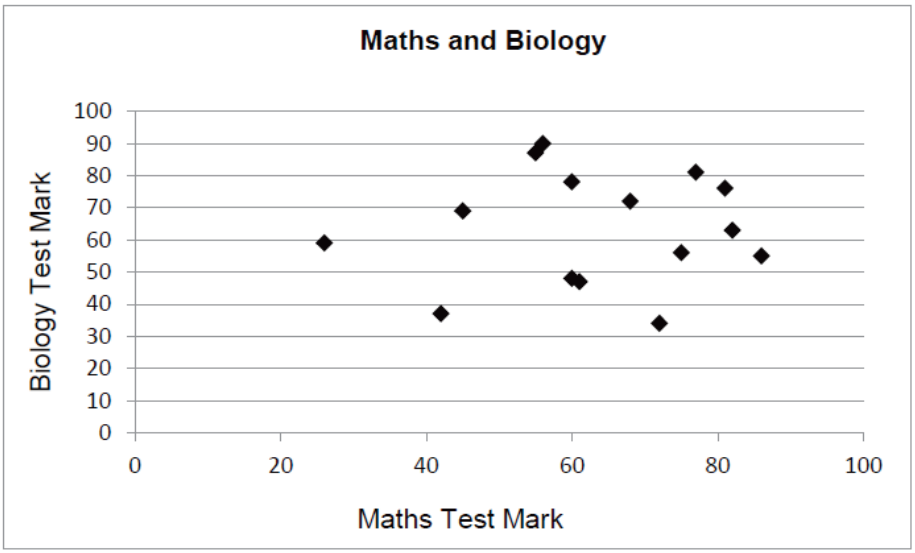
Solution
(i) Pie chart or column graph
(ii) students do not own bikes, not allowed to ride to school, older students tend not find it cool to ride, closer to walk, no security at school for bikes, students prefer to walk and talk
Specific behaviours
✓ identifies pie or column graph
✓ plausible reason given using list above as a guide

(c) Another student Libby, was investigating the relationship between students' marks in various subjects. Her results were displayed as follows.

Graph A: English and Biology test marks



Graph B: Maths and Biology test marks



Possible correlation coefficients: –0.8 0.79 –1 0.12 0.45

Which of the given correlation coefficients is closest to what Libby should get for:

(i) Graph A? (1 mark)

(ii) Graph B? (1 mark)

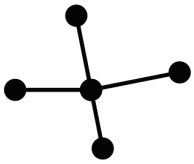
Solution
(i) Graph A: 0.79 (ii) Graph B: 0.12
Specific behaviours
✓ determines correct correlation for graph A ✓ determines correct correlation for graph B

Question 7 **(9 marks)**

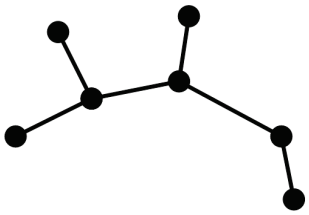
(a) (i) Complete the table for the following trees. (2 marks)

Tree A

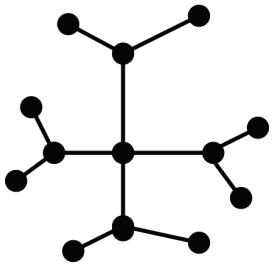
Tree B



Tree C



Tree D



Tree	A	B	C	D
Number of vertices (v)	6	5	7	13
Number of edges (e)	5	4	6	12

Solution
Completes table as shown above
Specific behaviours
✓ enters correctly all number of vertices ✓ enters correctly all number of edges

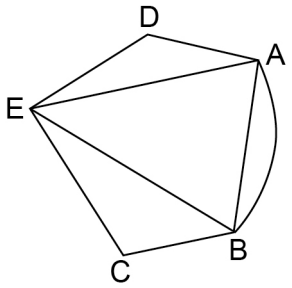
(ii) What appears to be the relationship between the number of vertices (v) and the number of edges (e) in the trees? (1 mark)

Solution
$v = e + 1$
Specific behaviours
✓ states correct equation or relationship

(b) All trees are planar. Why? (1 mark)

Solution
There is only one path between two vertices and therefore no crossing paths and can be drawn on the plane.
Specific behaviours
✓ states that planar graph can be drawn so no two paths cross

(c) Draw an adjacency matrix for the following network: (3 marks)



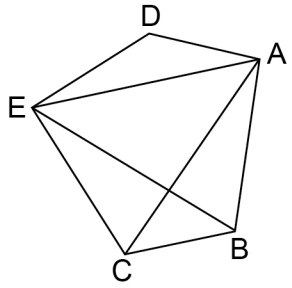
Solution

	A	**B**	**C**	**D**	**E**
A	0	2	0	1	1
B	2	0	1	0	1
C	0	1	0	0	1
D	1	0	0	0	1
E	1	1	1	1	0

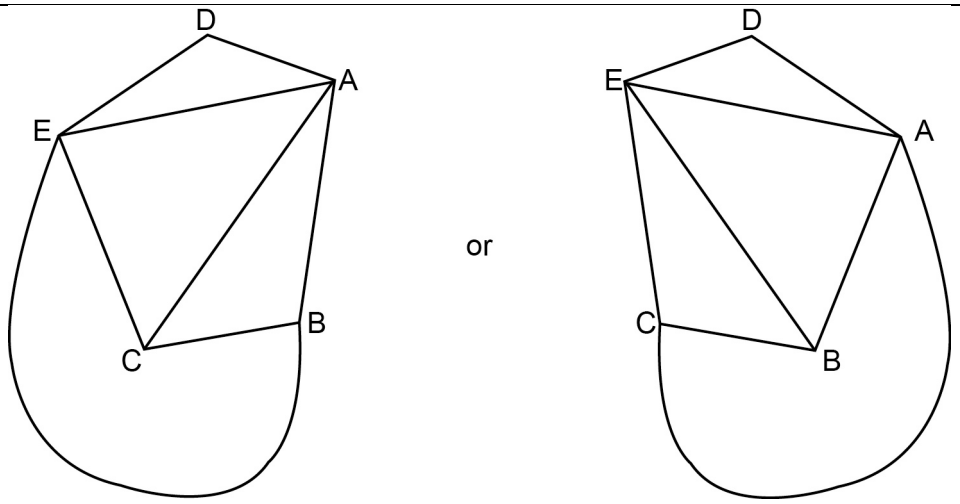
Specific behaviours

✓ identifies and draws a 5 × 5 adjacency matrix
✓ completes at least three rows correctly
✓ completes all rows and columns correctly

(d) Draw a planar version of the following network: (2 marks)



Solution



Specific behaviours

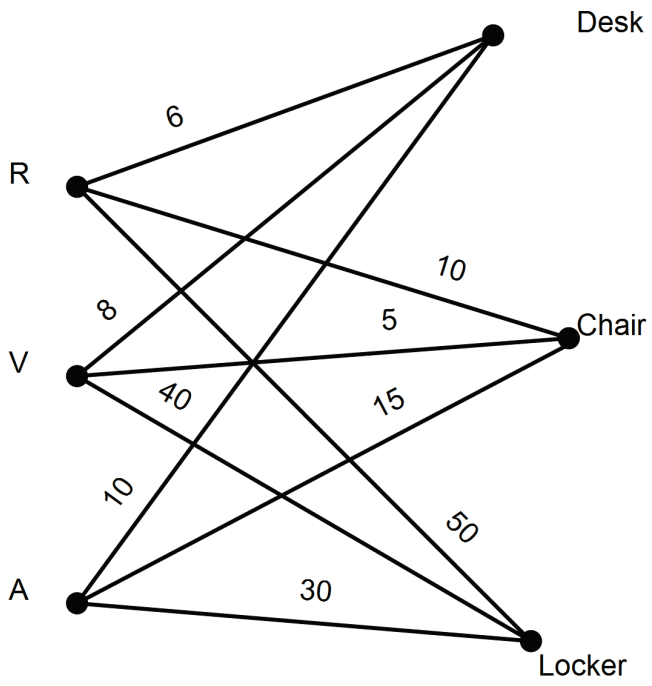
✓ redraws network to ensure that no two paths cross
✓ draws EB or AC outside the network

Question 8 **(5 marks)**

The following matrix shows the time (in minutes) taken for three students to complete three cleaning tasks. The rows in order represent the three students – Ruby, Vera and Andrew. The columns in order represent cleaning the desks, chairs and lockers.

$$\begin{bmatrix} 6 & 10 & 50 \\ 8 & 5 & 40 \\ 10 & 15 & 30 \end{bmatrix}$$

(a) Present this matrix as a labelled and weighted bipartite graph. (2 marks)

Solution

Specific behaviours
✓ correctly draws graph and vertices ✓ correctly labels edges

(b) Why is the graph called a complete bipartite graph? (1 mark)

Solution
all nodes in one group are connected to every node in the second group
Specific behaviours
✓ identifies that for a complete graph every node in one group is connected to every node in the other group

(c) How long would Ruby take to clean her desk, chair and locker? (1 mark)

Solution
66 minutes
Specific behaviours
✓ determines the time take correctly

(d) Who would take the shortest time to clean a chair? (1 mark)

Solution
Vera
Specific behaviours
✓ reads graph/matrix correctly to identify Vera takes the shortest time

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